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Chapter - 4

Cubes And Cube Roots.....

Introduction —

The natural number which is multiplied by itself three times is called perfect cube.

Ex - Let A be a natural number

$a \times a \times a = a^3$

Let A be - 3

$3 \times 3 \times 3 = 3^3$

$3^3 = 27$

Cube of Negative Integer

Ex - $(-a)^3 = -a \times -a \times -a$

Let A be -(-7)

$-7 \times -7 \times -7 = (-7)^3$

$(-7)^3 = -343$

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Cube of Rational Number

Ex- Let $\frac{a}{b}$ be a rational number

$$\frac{a \times a \times a}{b \times b \times b} = \frac{a^3}{b^3} = \left(\frac{a}{b}\right)^3$$

Let $\frac{a}{b}$ be $\frac{3}{5}$

$$\frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{3^3}{5^3} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

Try This ! (pg no 55)

2. Find the cubes of :-

a. 15
$= 15^3$
$= 15 \times 15 \times 15$
$= 3375$

b. 200
$= 200^3$
$= 200 \times 200 \times 200$
$= 8000000$

d. $\frac{1}{10}$
$= \left(\frac{1}{10}\right)^3$
$= \frac{1 \times 1 \times 1}{10 \times 10 \times 10}$

e. $\frac{-3}{4}$
$= \left(\frac{-3}{4}\right)^3$
$= \frac{-3 \times -3 \times -3}{4 \times 4 \times 4}$

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$= \frac{1}{1000}$ $= \frac{-27}{64}$

c. -6
$= (-6)^3$
$= -6 \times -6 \times -6$
$= -216$

---

3. Fill in the blanks :-

a. $(4a)^3 = \underline{64}\ a^3$

b. $(5x)^3 = \underline{125}\ x^3$

c. $\left(\frac{1}{2}t\right)^3 = \frac{1}{8}t^3$

d. $(\underline{5}\ y)^3 = 125\ y^3$

---

4. Evaluate :-

a. $\left(\sqrt{3^2+4^2}\right)^3$
$= \left(\sqrt{9+16}\right)^3$
$= \left(\sqrt{25}\right)^3$
$= 5^3$
$= 125$

b. $\left(\sqrt{6^2+8^2}\right)^3$
$= \left(\sqrt{36+64}\right)^3$
$= \left(\sqrt{100}\right)^3$
$= 10^3$
$= 1000$

c. $\left(\sqrt{40^2-24^2}\right)^3$
$= \left(\sqrt{1600-576}\right)^3$
$= \left(\sqrt{1024}\right)^3$
$= 32^3$
$= 32768$

d. $\left(\sqrt{17^2-15^2}\right)^3$
$= \left(\sqrt{289-225}\right)^3$
$= \left(\sqrt{64}\right)^3$
$= 8^3$
$= 512$

Excellent to see an attempt

Properties of Cubes

1. Cube of even numbers are also even
Ex- $2^3 = 8$, $4^3 = 64$, $6^3 = 216$, etc..

2. Cube of odd numbers are also odd.
Ex- $3^3 = 27$, $5^3 = 125$, $7^3 = 343$, etc..

3. Cube of a number ends with 0, 1, 4, 5, 6 & 9 also ends with the same digit

4. Cube of a number ends with 2, 8, 3 & 7 do not ends with the same digit.

TABLE :-

- Cube of number with last digit 1 ends with $1 - 11^3 = 1331$.
- Cube of number with last digit 2 ends with $8 - 12^3 = 1728$.
- Cube of number with last digit 3 ends with $7 - 13^3 = 2197$.
- Cube of number with last digit 4 ends with $4 - 14^3 = 2744$.
- Cube of number with last digit 5 ends with $5 - 15^3 = 3375$.
- Cube of number with last digit 6 ends with $6 - 16^3 = 4096$.
- Cube of number with last digit 7 ends with $3 - 17^3 = 4913$.
- Cube of number with last digit 8 ends with $2 - 18^3 = 5832$.
- Cube of number with last digit 9 ends with $9 - 19^3 = 6859$.
- Cube of number with last digit 0 ends with $0 - 20^3 = 8000$.

5. # $2^3 = 8$
$20^3 = 8 \times 10^3$
$\downarrow$
$[2 \times 10^3] = 2^3 \times 10^3$
$= 8 \times 10^3$
$200 = 8 \times 10^6$
$2000 = 8 \times 10^9$
$20000 = 8 \times 10^{12}$

$3^3 = 27$
$30^3 = 27 \times 10^3$
$\downarrow$
$[3 \times 10^3] = 3^3 \times 10^3$
$= 27 \times 10^3$
$300 = 27 \times 10^6$
$3000 = 27 \times 10^9$
$30000 = 27 \times 10^{12}$

Some Interesting Facts

- We can note that by placing odd numbers in the form of a triangle and then adding the rows we get cube number.

$1 = 1^3 = 1$

$3+5 = 2^3 = 8$

$7+9+11 = 3^3 = 27$

$13+15+17+19 = 4^3 = 64$

$21+23+25+27+29 = 5^3 = 125$

- Difference of Cubes of two Consecutive Number

Let n be a Number

n+1 be a consecutive number, then -

$(n+1)^3 - (n)^3 = n^3 + 1^3 + 3 \times n \times 1 (n+1) - n^3$

$= 1 + 3n(n+1)$

$= 1 + 3n^2 + 3n$

$(n+1)^3 - (n)^3 = 1 + 3n^2 + 3n$

Let n be - 13

$n+1 = 13+1 = 14$

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$(13+1)^3 - 13^3 = 3 \times 13^2 + 3 \times 13 + 1$

$14^3 - 13^3 = 3 \times 169 + 39 + 1$

$14^3 - 13^3 = 507 + 40$

$14^3 - 13^3 = 547$

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Try This ! (Pg no. 56)

1. Find the ones digit of the cube of the following numbers.

a. 81
= Ones digit = 1

b. 37
= Ones digit = 3

c. 65
= Ones digit = 5

d. 113
= Ones digit = 7

e. 244
= Ones digit = 4

f. 179
= Ones digit = 9

g. 222
= Ones digit = 8

h. 456
= Ones digit = 6

2. Find which of the following numbers have even numbers as their cubes and which will have odd numbers as their cubes.

a. 243
= 243 is odd
= Thus, 243^3

b. 559
= 559 is odd
= Thus, 559^3 is odd

c. 822
= 822 is even
= Thus, 822^3 is even

d. 98
= 98 is even
= Thus, 98^3 is even

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3. Which of the following are perfect cubes?

a. 800
= $2^3 \times 10^2$
= So, It is not a perfect cube

b. 1331000
= $11^3 \times 10^3$
= So, It is a perfect cube

c. 125000
= $5^3 \times 10^3$
= So, It is a perfect cube

d. 16000
= $4^2 \times 10^3$
= So, It is not a perfect cube

---

4. Express 8^3 as a sum of odd natural numbers
= $57+59+61+63+65+67+69+71 = 8^3 = 512$

---

5. Evaluate without finding actual cubes:

a. $11^3 - 10^3$
= $(n+1)^3 - n^3 = 3n^2 + 3n + 1$
= $(10+1)^3 - 10^3 = 3 \times 10^2 + 3 \times 10 + 1$
= $3 \times 100 + 30 + 1$
= $300 + 31$
= 331

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6. $14^3 - 13^3$
= $(n+1)^3 - n^3 = 3n^2 + 3n + 1$
= $(13+1)^3 - 13^3 = 3 \times 13^2 + 3 \times 13 + 1$
= $3 \times 169 + 39 + 1$
= $507 + 40$
= 547

Exercise - 4.1

1. Find the cubes of:

a. 9
= 9^3
= $9 \times 9 \times 9$
= 729

b. 300
= 300^3
= $300 \times 300 \times 300$
= 27000000

c. $\frac{-6}{7}$
= $\left[\frac{-6}{7}\right]^3$
= $\frac{-6 \times -6 \times -6}{7 \times 7 \times 7}$
= $\frac{-216}{343}$

d. $\frac{-3}{10}$
= $\left[\frac{-3}{10}\right]^3$
= $\frac{-3 \times -3 \times -3}{10 \times 10 \times 10}$
= $\frac{-27}{1000}$

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e. 13
$= 13^3$
$= 13 \times 13 \times 13$
$= 2197$

f. 2000
$= 2000^3$
$= 2000 \times 2000 \times 2000$
$= 8000000000$

g. $\frac{-4}{9}$
$= \left(\frac{-4}{9}\right)^3$
$= \frac{-4 \times -4 \times -4}{9 \times 9 \times 9}$
$= \frac{-64}{729}$

h. $\frac{-8}{100}$
$= \left(\frac{-8}{100}\right)^3$
$= \frac{-8 \times -8 \times -8}{100 \times 100 \times 100}$
$= \frac{-512}{1000000}$

2. Find by what number the cube of a number gets multiplied if the number is multiplied by:

a. 3
$= 3 \times 3 \times 3$, 9 no
$= 27$

b. 5
$= 5 \times 5 \times 5$, 25 no.
$= 125$

3. Evaluate:

a. $(\sqrt{5^2 + 12^2})^3$
$= (\sqrt{25 + 144})^3$
$= (\sqrt{169})^3$
$= 13^3$
$= 2197$

b. $(\sqrt{26^2 - 10^2})^3$
$= (\sqrt{676 - 100})^3$
$= (\sqrt{576})^3$
$= 24^3$
$= 13824$

c. $(\sqrt{61^2 - 60^2})^3$
$= (\sqrt{3721 - 3600})^3$
$= (\sqrt{121})^3$
$= 11^3$
$= 1331$

d. $(\sqrt{24^2 + 7^2})^3$
$= (\sqrt{576 + 49})^3$
$= (\sqrt{625})^3$
$= 25^3$
$=$

e. $(\sqrt{50^2 - 14^2})^3$
$= (\sqrt{2500 - 196})^3$
$= (\sqrt{2304})^3$
$= 48^3$
$= 110592$

f. $(\sqrt{85^2 - 84^2})^3$
$= (\sqrt{7225 - 7056})^3$
$= (\sqrt{169})^3$
$= 13^3$
$= 2197$

4. Write 'even' against those whose cubes are even and 'odd' against those whose cubes are odd.

a. 3426 : Even
b. 28755 : Odd
c. 2422 : Even
d. 6875 : Odd

5. Without actually finding the cube, find the digit at the ones place of cubes of the following numbers.

a. 72
= Ones digit → 8

b. 91
= Ones digit → 1

c. 60
= Ones digit → 0

d. 124
= Ones digit → 4

e. 98
= Ones digit → 2

f. 83
= Ones digit → 7

g. 79
= Ones digit → 9

h. 36
= Ones digit → 6

5. Identify the numbers which are perfect cubes:

a. 1600
$= 4^2 \times 10^2$
= It is not a perfect cube.

b. 216000
$= 6^3 \times 10^3$
= It is a perfect cube.

c. $\frac{1}{27000}$
$= \frac{1}{3^3 \times 10^3}$
= It is a perfect cube.

d. 729000
$= 9^3 \times 10^2$
= It is not a perfect cube.

e. 6400
$= 4^3 \times 10^2$
= It is not a perfect cube.

f. $\frac{1}{8100}$
$= \frac{1}{9^2 \times 10^2}$
= It is not a perfect cube.

7. (a) Write 6^3 as sum of consecutive odd numbers.
$= 31 + 33 + 35 + 37 + 39 + 41 = 6^3 = 216$

(b) Write 9^3 as sum of consecutive odd numbers
$= 73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89 = 9^3 = 729$

8. Without finding the cubes, evaluate the following:

a. $13^3 - 12^3$
$= (n+1)^3 - n^3 = 3n^2 + 3n + 1$
$= (12+1)^3 - 12^3 = 3 \times 12^2 + 3 \times 12 + 1$
$= 3 \times 144 + 36 + 1$
$= 432 + 37$
$= 469$

b. $8^3 - 7^3$
$= (n+1)^3 - n^3 = 3n^2 + 3n + 1$
$= (7+1)^3 - 7^3 = 3 \times 7^2 + 3 \times 7 + 1$
$= 3 \times 49 + 21 + 1$
$= 147 + 22$
$= 169$

c. $101^3 - 100^3$

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$(100+1)^3 - 100^3 = 3 \times 100^2 + 3 \times 100 + 1$

$= 3 \times 10000 + 300 + 1$

$= 30000 + 301$

$= 30301$

d. $9^3 - 8^3$

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$(8+1)^3 - 8^3 = 3 \times 8^2 + 3 \times 8 + 1$

$= 3 \times 64 + 24 + 1$

$= 192 + 25$

$= 217$

e. $12^3 - 11^3$

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$(11+1)^3 - 11^3 = 3 \times 11^2 + 3 \times 11 + 1$

$= 3 \times 121 + 33 + 1$

$= 363 + 34$

$= 397$

f. $51^3 - 50^3$

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$(50+1)^3 - 50^3 = 3 \times 50^2 + 3 \times 50 + 1$

$= 3 \times 2500 + 150 + 1$

$= 7500 + 151$

$= 7651$

Excellent

Cube Root Of A Number

Cube Root of a Natural Number -

Let A be a number

Then -

$\sqrt[3]{a} = \sqrt[3]{6 \times 6 \times 6 \ldots}$

$\sqrt[3]{a} = \sqrt[3]{b^3}$

$\sqrt[3]{a} = b$

Ex — $\sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$\sqrt[3]{216} = \sqrt[3]{2^3 \times 3^3}$

$\sqrt[3]{216} = 2 \times 3$

$\sqrt[3]{216} = 6$

Cube Root of a Negative Number -

Let A be a number

Then -

$\sqrt[3]{-a} = -\sqrt[3]{a}$

Ex — $\sqrt[3]{-125} = -\sqrt[3]{125}$

$\sqrt[3]{-125} = -\sqrt[3]{5 \times 5 \times 5}$

$\sqrt[3]{-125} = -\sqrt[3]{5^3}$

$\sqrt[3]{-125} = -5$

Cube Root of a Rational Number -

Let $\frac{a}{b}$ be a rational number

Then-

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Ex- $\sqrt[3]{\frac{343}{512}} = \frac{\sqrt[3]{343}}{\sqrt[3]{512}}$

$= \frac{\sqrt[3]{7\times7\times7}}{\sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2}}$

$= \frac{\sqrt[3]{7^3}}{\sqrt[3]{2^3\times2^3\times2^3}}$

$= \frac{7}{2\times2\times2}$

$= \frac{7}{8}$

- Find the smallest number by which 3087 be divided and multiplied so that the number will be perfect cube. Also, find the cube root of the number thus obtained.

Multiply:

$3087 = 3\times3\times7\times7\times7$

$= 3^2\times7^3$

3 is needed for pairing

So, 3 is the smallest number by which 3087 be multiplied to get a new perfect cube number.

The new perfect cube number $= 3087\times3$

$= 9261$

$\sqrt[3]{9261} = \sqrt[3]{3\times3\times3\times7\times7\times7}$

$= \sqrt[3]{3^3\times7^3}$

$= 3\times7$

$= 21$

Case II: Divide:

$3\times3 = 9$

So, 9 is the smallest number by which 3087 be divided to get a new perfect cube number.

The new perfect cube number $= 3087 \div 9$

$= 343$

$\sqrt[3]{343} = \sqrt[3]{7\times7\times7}$

$= \sqrt[3]{7^3}$

$= 7^3$

Try This ! (pg no. 59)

1. Find the cube root by prime factorisation method.

b. -343

7	343
7	49
7	7
	1

$-343 = -7\times7\times7$

$-343 = -7^3$

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a. 10,00,000

2	1000000
2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$1000000 = 2\times2\times2\times2\times2\times2\times5\times5\times5\times5\times5\times5$

$1000000 = 2^3 \times 2^3 \times 5^3 \times 5^3$

$1000000 = 100^3$

c. $\frac{1000}{2197}$

2	1000
2	500
2	250
5	125
5	25
5	5
	1

13	2197
13	169
13	13
	1

$\frac{1000}{2197} = \frac{2\times2\times2\times5\times5\times5}{13\times13\times13}$

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$\frac{1000}{2197} = \frac{2^3\times5^3}{13^3}$

$\frac{1000}{2197} = \frac{10^3}{13^3}$

d. $\frac{-1}{64}$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = 2\times2\times2\times2\times2\times2$

$64 = 2^3 \times 2^3$

$64 = 4^3$

$\frac{-1}{64} = \left(\frac{-1}{4}\right)^3$

h. 42875

5	42875
5	8775
5	1715
7	343
7	49
7	7
	1

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$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$42875 = 5^3 \times 7^3$

$42875 = 35^3$

f. 46656

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$46656 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$46656 = 2^3 \times 2^3 \times 3^3 \times 3^3$

$46656 = 36^3$

g. 250047

$250047 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$250047 = 3^3 \times 3^3 \times 7^3$

$250047 = 63^3$

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3	250047
3	83349
3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

e. 74088

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$74088 = 2^3 \times 3^3 \times 7^3$

$74088 = 42^3$

2. Find the smallest number by which : [Also, find the cube root of the number thus obtained.]

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a. 1323 must be multiplied so that the product is a perfect cube

3	1323
3	441
3	147
7	49
7	7
	1

$1323 = 3 \times 3 \times 3 \times 7 \times 7$
$= 3^3 \times 7^2$

7 is needed for pairing Ans – ①

So, 7 is smallest number that must be multiplied by 1323 to make it a perfect cube no.

The new perfect cube number $= 1323 \times 7$
$= 9261$

$\sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$
$= \sqrt[3]{3^3 \times 7^3}$
$= 3 \times 7$
$= 21$ Ans – ②

b. 8640 must be divided so that the quotient is a cube number.

5 is the number for pairing. Ans – ①

So, 5 is the smallest number that must be divided by 8640 to make it a perfect cube no.

The new perfect cube number $= 8640 \div 5$
$= 1728$

$\sqrt[3]{1728} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$
$= \sqrt[3]{2^3 \times 2^3 \times 3^3}$
$= 2 \times 2 \times 3$
$= 12$ Ans – ②

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2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

$8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^3 \times 2^3 \times 3^3 \times 5$

3. Evaluate:

a. $\sqrt[3]{\frac{125}{2744}}$
$= \frac{5 \times 5 \times 5}{2 \times 2 \times 2 \times 7 \times 7 \times 7}$
$= \frac{5^3}{2^3 \times 7^3}$
$= \left[\frac{5}{2 \times 7}\right]^3$
$= \left[\frac{5}{14}\right]^3$

b. $\sqrt[3]{\frac{-64}{35937}}$
$= \frac{-4 \times -4 \times -4}{3 \times 3 \times 3 \times 11 \times 11 \times 11}$
$= \frac{-4^3}{3^3 \times 11^3}$
$= \left[\frac{-4}{3 \times 11}\right]^3$
$= \left[\frac{-4}{33}\right]^3$

c. $\sqrt[3]{\frac{27000}{343}}$
$= \frac{3 \times 3 \times 3 \times 10 \times 10 \times 10}{7 \times 7 \times 7}$
$= \frac{3^3 \times 10^3}{7^3}$
$= \left[\frac{3 \times 10}{7}\right]^3$
$= \left[\frac{30}{7}\right]^3$

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4. Three numbers are in ratio 1:2:3. If the sum of their cubes is 2304, find the three numbers.

Ratio of numbers = 1:2:3

Sum of their cubes = 2304

Let the 3 numbers = $x, 2x, 3x$

ATQ ⇒

$$(x)^3 + (2x)^3 + (3x)^3 = 2304$$
$$x^3 + 8x^3 + 27x^3 = 2304$$
$$36x^3 = 2304$$
$$x^3 = \frac{2304}{36}$$
$$x^3 = 64$$

$$x^3 = 64$$
$$x = \sqrt[3]{64}$$
$$x = 4$$

Hence,

1st number = x = 4

2nd number = $2x = 2 \times 4 = 8$

3rd number = $3x = 3 \times 4 = 12$

1st number = 4
2nd number = 8
3rd number = 12 Ans.

5. Suman made a cuboid out of plasticine measuring 2 cm by 6 cm by 9 cm. How many such cuboids are required to make a perfect cube?

Volume of cuboid = L × B × H
= 2 × 6 × 9 cm
= 108 cm

$\sqrt[3]{108} = \sqrt[3]{2 \times 2 \times 3 \times 3 \times 3}$
$= \sqrt[3]{2^2 \times 3^3}$
= 2 is required for making 108 a perfect cube. Ans

6. Find the ones digit of the cube roots of following cube numbers.

a. 2197
= Ones digit = 3

b. 19,683
= Ones digit = 7

c. 27,000
= Ones digit = 0

d. 91,125
= Ones digit = 5

w. fair & excellent

Cube Root By Estimation Method

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Ex- 912673

912 → Group 2nd

673 → Group 1st

$9^3 = 729$

$10^3 = 1000$

$3^3 = 27$

Ones digit = 7

$729 < 912 < 1000$

912 is nearest to 729 (9^3)

Hence, $\sqrt[3]{912673} = 97$ Ans

Try This ! (pg no. 61)

1. Estimate the cube roots of the following.

a. 551368

= 1st Group = 368, Ones digit $[8^3 = 512] = 2$

= 2nd Group = 551, $512 < 551 < 729$

= 551 is nearest to 512 (8^3)

= Hence, $\sqrt[3]{551368} = 82$ Ans

b. 166375

= 1st Group = 375, Ones digit $[5^3 = 125] = 5$

= 2nd Group = 166, $125 < 166 < 216$

= 166 is nearest to 125 (5^3)

= Hence, $\sqrt[3]{166375} = 55$ Ans

Excellent