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CHAPTER – 2

Polynomial

★ The expression that are in the form of $a_0x^0 +$ $a_1x^1 + a_2x^2 \ldots\ldots a_nx^n$ where n be a +ve positive integer is called a polynomial.

Examples :–

a) $(2x+3)$, degree = 1, Linear polynomial.
b) x^2+3, degree = 2, Quadratic polynomial.
c) x^3+x, degree = 3, Cubic polynomial.
d) x^4+1, degree = 4, Biquadratic polynomial.
e) 15, degree = 0 as $15 \times x^0 = 15 \times 1$, Constant polynomial.
f) 0, degree = not defined, Zero polynomial.

☆ Value of Polynomial at a Point

	Standard
Eg- $p(x) = x+3$ $x = 1$ value at $x=1$ $p(1) = 1+3$ $= 4$	$p(x)$, where value at point $x = \alpha$ value $= p(\alpha)$

Zero of Polynomial :–

$p(x)$ at point $x = \alpha$

If value of $\boxed{p(\alpha) = 0}$ then, α is zero of polynomial.

Eg. $p(x) = x-1$
$p(1) = 1-1$
$= 0$ then, 0 is zero of $p(x)$

Note → No of zero's = degree of polynomial.

Relation between Zeros & coefficient

1) Quadratic ⇒ eg - $ax^2 + bx + c$
zeros are α, β

i) Sum of zero = $-\dfrac{\text{Coeff. of } x}{\text{coeff of } x^2}$

$\Rightarrow \alpha + \beta = \dfrac{-b}{a}$

ii) Product = $\dfrac{\text{Constant}}{\text{coeff. of } x^2}$

$\Rightarrow \alpha\beta = \dfrac{c}{a}$

2) Cubic ⇒ eg $ax^3 + bx^2 + cx + d$
α, β & γ are ~~ze~~ zeros

i) Sum of zeros = $-\dfrac{\text{coeff. of } x^2}{\text{coeff. of } x^3}$

$\Rightarrow \alpha + \beta + \gamma = -\dfrac{b}{a}$

ii) Sum taking product of two at a time = $\dfrac{\text{coeff of } x}{\text{coeff of } x^3}$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

iii) Product = $\dfrac{\text{Constant}}{\text{coeff. of } x^3}$

$\Rightarrow \alpha\beta\gamma = \dfrac{-d}{a}$

Questions ⟨Ex - 2.2⟩

Find zeroes & verify the relationship between zeroes & coefficient :-

i) $x^2 - 2x - 8$

$\Rightarrow x^2 - (4-2)x - 8$

$\Rightarrow x^2 - 4x + 2x - 8$

$\Rightarrow x(x-4) + 2(x-4)$

$\Rightarrow (x-4)(x+2) = 0$

$\underset{\alpha}{x = 4}$, $\underset{\beta}{x = -2}$

$\rightarrow \{ax^2 + bx + c\}$

$a = 1$, $b = -2$, $c = -8$

$\Rightarrow \alpha + \beta = -\dfrac{b}{a}$

$\Rightarrow \alpha + \beta = -\dfrac{(-2)}{1} = 2$

$\Rightarrow \alpha \times \beta = \dfrac{c}{a}$

$\Rightarrow \alpha \times \beta = \dfrac{-8}{1} = -8$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
$= \sqrt{4 - 4(-8)}$
$= \sqrt{4 + 32}$
$= \sqrt{36}$
$\alpha - \beta = 6$

$\alpha + \beta = 2$
$\alpha - \beta = 6$
$2\alpha = 8$
$\boxed{\alpha = 4}$
$\alpha - \beta = -8$
$\boxed{\beta = -2}$

$\therefore (-2)^2 - 2 \times (-2) - 8$
$= 4 + 4 - 8$
$= 8 - 8$
$= 0$

& $4^2 - 2(4) - 8$
$= 16 - 8 - 8$
$= 16 - 16$
$= 0$

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ii) $4s^2-4s+1$
$\rightarrow 4s^2-(2+2)s+1$
$\rightarrow 4s^2-2s-2s+1$
$\rightarrow 2s(2s-1)-1(2s-1)$
$\rightarrow (2s-1)(2s-1)$
$\alpha = \beta = \frac{1}{2}$

here, as^2+bs+c
$a=4,\ b=-4,\ c=1$

i) $\alpha+\beta = \frac{-b}{a}$

$\frac{1}{2}+\frac{1}{2} = \frac{4}{4}$

$\frac{\cancel{2}}{\cancel{2}} = \frac{4}{4}$

$1 = 1$

ii) $\alpha\cdot\beta = \frac{c}{a}$

$\Rightarrow \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

$\Rightarrow \frac{1}{4} = \frac{1}{4}$
verified

iii) $4u^2+8u$
$\Rightarrow 4u(u+2)$
$\Rightarrow 4u=0,\ u+2=0$
$\Rightarrow u=0=\alpha,\ u=-2=\beta$

$\therefore au^2+bu+c$
where $a=4,\ b=8,\ c=0$

i) $\alpha+\beta = \frac{-b}{a}$

$\Rightarrow -2 = \frac{-8}{4}$

$\Rightarrow -2 = -2$

ii) $\alpha\cdot\beta = \frac{c}{a}$

$\Rightarrow 0 = \frac{0}{4}$

$\Rightarrow 0 = 0$ verified.

iii) t^2-15
$\rightarrow t^2-15=0$
$\Rightarrow t^2=15$
$\Rightarrow t=\pm\sqrt{15}$
$\alpha=\sqrt{15}\quad \beta=\sqrt{15}$

$\therefore at^2+bt+c$ where $a=1,\ b=0,\ c=-15$

i) $\alpha+\beta = \frac{-b}{a}$

$\Rightarrow 0=0$

ii) $\alpha\cdot\beta = \frac{c}{a}$

$\Rightarrow \sqrt{15}\times-\sqrt{15} = \frac{-15}{1}$

$\Rightarrow -15 = -15$ verified.

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iv) ~~$3x^2+3=7x$~~ ~~x^2+3-7x~~
$\Rightarrow$ ~~x^2-7x-3~~
$\Rightarrow$ ~~x^2-~~

iv) $x^2+7x+12$
$\Rightarrow x^2+(4+3)x+12$
$\Rightarrow x(x+4)+3(x+4)$
$\Rightarrow (x+4)(x+3)$

For zeroes, $x=-4,\ x=-3$
here $a=1,\ b=7,\ c=12$

i) $\alpha+\beta = \frac{-7}{1}$

$-4-3=-7$
$-7=-7$
verified

ii) $\alpha\cdot\beta = \frac{c}{a}$

$(-3)(-4) = \frac{12}{1}$

$12=12$
LHS = RHS

v) $6x^2-7x-3$
$\Rightarrow 6x^2-(9-2)x-3$
$\Rightarrow 6x^2-9x+2x-3$
$\Rightarrow 3x(2x-3)+1(2x-3)$
$\Rightarrow \underset{\alpha}{(2x-3)}\underset{\beta}{(3x+1)}$

$x=\frac{3}{2},\ x=\frac{-1}{3}$

i) $\frac{3}{2}+\left(\frac{-1}{3}\right) = \frac{-(-7)}{6}$

$\Rightarrow \frac{9-2}{6} = \frac{7}{6}$

$\Rightarrow \frac{7}{6} = \frac{7}{6}$
LHS = RHS

ii) $\frac{\cancel{3}}{2}\times\frac{(-1)}{\cancel{3}} = \frac{-3}{6\,2}$

$\Rightarrow \frac{-1}{2} = \frac{-1}{2}$
Verified

vi) $4\sqrt{3}x^2+5x-2\sqrt{3}$
$\Rightarrow 4\sqrt{3}x^2+(8-3)x-2\sqrt{3}$
$\Rightarrow 4\sqrt{3}x^2+8x-3x-2\sqrt{3}$
$\Rightarrow 4\sqrt{3}x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$
$\Rightarrow (\sqrt{3}x+2)(4x-\sqrt{3})$

For zeroes, $x=\frac{-2}{\sqrt{3}}$ (α), $x=\frac{\sqrt{3}}{4}$ (β)

here $a=4\sqrt{3},\ b=5,\ c=-2\sqrt{3}$

i) Sum $= \frac{-b}{a}$

$\Rightarrow \frac{-2\sqrt{3}}{\sqrt{3}}+\frac{\sqrt{3}}{4} = \frac{-5}{4\sqrt{3}}$

$\Rightarrow \frac{-5}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}}$
LHS = R.H.S

ii) Product $= \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{-2\sqrt{3}}{4\sqrt{3}}$

$\Rightarrow \frac{-1}{2} = \frac{-1}{2}$ verified

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(2) Find quadratic polynomial - - - - - - respectively

(i) $\frac{1}{4}, -1$

eq^n :— $x^2 - (\alpha+\beta)x + \alpha\cdot\beta$ $\{\alpha+\beta = \frac{1}{4}, \alpha\cdot\beta = -1\}$

$\Rightarrow x^2 - \frac{1}{4}x - 1 = 0$

$\Rightarrow \frac{4x^2 - x - 4}{4} = 0$

$\Rightarrow 4x^2 - x - 4$

ii) $\sqrt{2}, \frac{1}{3}$

$\Rightarrow$ eq :- $x^2 - (\alpha+\beta)x + \alpha\cdot\beta$ $\{\alpha+\beta = \sqrt{2}, \alpha\cdot\beta = \frac{1}{3}\}$

$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$

$\Rightarrow \cancel{x^2}\ \frac{3x^2 - 3x\sqrt{2} + 1}{3} = 0$

$\Rightarrow 3x^2 - 3x\sqrt{2} + 1$

iii) $0, \sqrt{5}$

$\Rightarrow eq^n$:- $x^2 - (\alpha+\beta)x + \alpha\cdot\beta$ $\{\alpha+\beta = 0, \alpha\cdot\beta = \sqrt{5}\}$

$\Rightarrow x^2 - 0 + \sqrt{5}$

$\Rightarrow x^2 + \sqrt{5}$

iv) $1, 1$

$\Rightarrow eq^n$:- $x^2 - (\alpha+\beta)x + \alpha\beta$ $\{\alpha+\beta = 1, \alpha\cdot\beta = 1\}$

$\Rightarrow x^2 - x + 1$

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v) $-\frac{1}{4}, \frac{1}{4}$

$\Rightarrow eq^n$:— $x^2 - (\alpha+\beta)x + \alpha\cdot\beta$ $(\alpha+\beta = -\frac{1}{4}, \alpha\cdot\beta = \frac{1}{4})$

$\Rightarrow x^2 - \frac{x}{4} + \frac{1}{4} = 0$

$\Rightarrow \frac{4x^2 - x + 1}{4} = 0$

$\Rightarrow 4x^2 - x + 1$

vi) $4, 1$

$\Rightarrow eq^n$:— $x^2 - (\alpha+\beta)x + \alpha\cdot\beta$ $(\alpha+\beta = 4, \alpha\cdot\beta = 1)$

$\Rightarrow x^2 - 4x + 1$

Division Algorithm

$f(x) = g(x)\cdot q(x) + r(x)$
where, degree of $r(x) = 0 <$ degree of $g(x)$

25/04/19

Exercise - 2.3

1. Divide - - - - - - - of the following :-

i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

$\Rightarrow$

$x^2 - 2\ )\ x^3 - 3x^2 + 5x - 3\ (\ x - 3$
$\quad\quad x^3 \quad\quad \mp 2x$
$\quad\quad -3x^2 + 7x - 3$
$\quad\quad \mp 3x^2 \quad \pm 6$
$\quad\quad 7x - 9$

quotient $= x - 8$ & remainder $= 7x - 9$

ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

$\Rightarrow$ $x^2+1-x \,\overline{)\, x^4 - 3x^2 + 4x + 5 \,}\,(x^2 + x - 3$

$\cancel{x^4} \mp x^2 \quad \pm x^3$

$+x^3 - 4x^2 + 4x + 5$

$\cancel{x^3} \mp x^2 \pm x$

$-3x^2 + 3x + 5$

$-3x^2 \pm 3x \mp 15$

20

quotient $= x^2 + x - 3$, remainder $= 20$

2. Check whether the first - - - - - - - polynomial.

i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$\Rightarrow$ $t^2-3 \,\overline{)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12 \,}\,(2t^2 + 3t + 4$

$\cancel{2t^4} \mp 6t^2$

$3t^3 + 4t^2 - 9t - 12$

$\cancel{3t^3} \mp 9t$

$4t^2 - 12$

$4t^2 - 12$

0 Yes

iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$\Rightarrow$ $x^3-3x+1 \,\overline{)\, x^5 - 4x^3 + x^2 + 3x + 1 \,}\,(x^2 - 1$

$\cancel{x^5} \mp 3x^3 + x^2$

$-x^3 + 3x + 1$

$-x^3 + 3x \mp 1$

2 NO

3. Obtain - - - - - - - - - - - - - $\sqrt{\frac{5}{3}}$ & $-\sqrt{\frac{5}{3}}$

$\Rightarrow$ Let $p(x) = (3x^4 + 6x^3 - 2x^2 - 10x - 5)$

zeroes $= \sqrt{\frac{5}{3}}$ & $-\sqrt{\frac{5}{3}}$

$\left(x - \sqrt{\frac{5}{3}}\right) \cdot \left[x - \left(-\sqrt{\frac{5}{3}}\right)\right]$ are factors of $p(x)$

$\Rightarrow \therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) \Rightarrow x^2 - \left(\sqrt{\frac{5}{3}}\right)^2$

$\Rightarrow x^2 - \frac{5}{3}$

$\Rightarrow \frac{3x^2 - 5}{3} \Rightarrow \frac{1}{3}(3x^2 - 5)$

so, $(3x^2 - 5)$ is factor of $p(x)$

$3x^2-5 \,\overline{)\, 3x^4 + 6x^3 - 2x^2 - 10x - 5 \,}\,(x^2 + 2x + 1$

$\cancel{3x^4} \mp 5x^2$

$6x^3 + 3x^2 - 10x - 5$

$\cancel{6x^3} \mp 10x$

$3x^2 - 5$

$3x^2 \mp 5$

0

$p(x) = (3x^2 - 5)(x^2 + 2x + 1)$

$= (3x^2 - 5)(x + 1)^2$

for zero's as $x+1$ is square

$x = -1$ & $x = -1$

-1 & -1 are the ~~rest~~ zeroes of polynomial $p(x)$.

4. On dividing - - - - - - - - - - - - - $g(x)$.

$\Rightarrow$ $p(x) = x^3 - 3x^2 + x + 2$, quotient $= x-2$ &
remainder $= -2x+4$.

$\Rightarrow$ $p(x) = g(x)\,(x-2) + (-2x+4)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 = g(x)\,(x-2) + (-2x+4)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$

$\Rightarrow$ $g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x-2}$

Hence $g(x) = x^2 - x + 1$

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