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Unit - 4 ⇒ QUADRATIC EQUATIONS

1\. i) $(x+1)^2 = 2(x-3)$

$\Rightarrow (a+b)^2 = a^2+b^2+2ab$

$x^2+2x+1 = 2x-6$

$x^2+2x+1-2x+6 = 0$

$\Rightarrow x^2+7 = 0$

→ degree of the eq. ⇒ 2

Equation is in the form of $ax^2+bx+c=0$ where $a \neq 0$

So, it is quadratic eq.

ii) $x^2-2x = (-2)(3-x)$

$x^2-2x = -6+2x$

$x^2-2x+6-2x = 0$

$x^2-4x+6 = 0$

⇒ eq. is in the form of $ax^2+bx+c=0$ with $a \neq 0$

∴ It is a quadratic eq.

iii) $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x^2+x-2x-2 = x^2+3x-x-3$

$x^2-x-2 = x^2+2x-3$

$x^2-x-2-x^2-2x+3 = 0$

$-3x+1 = 0$

$3x-1 = 0$

∴ It is not a quadratic eq.

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vii) $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow (a+b)^3 = a^3+b^3+3ab(a^2+b^2)$

$x^3+8+6x^2+12x = 2x^3-2x$

$x^3+8+6x^2+12x-2x^3+2x=0$

$-x^3+8+6x^2+14x=0$

$x^3-14x-6x^2-8=0$

$\therefore$ The given eq. is not a quadratic eq.

viii) $x^3-4x^2-x+1=(x-2)^3$

$\Rightarrow (a-b)^3 = a^3-b^3-3a^2b+3ab^2$

$x^3-4x^2-x+1 = x^3-8-6x^2+12x$

$x^3-4x^2-x+1-x^3+8+6x^2-12x=0$

$2x^2-13x+9=0$

$\therefore$ The given eq. is a quadratic eq.

2. i) Let the breadth of the plot be x m.

ACQ,

length of plot $= (2x+1)$m

Area of rectangle $= l \times b$

$\therefore 528 = x(2x+1)$

$2x^2+x-528=0$ (Required quadratic form).

2. Let the consecutive integer be x and $x+1$.

$\therefore$ Their product $= 306$

$\therefore x(x+1) = 306$

$x^2+x-306=0$ (required quadratic form)

iii) Let Rohan's age be x.

his mother's age $= x+26$

3 years hence;

Rohan's age $= x+3$

Mother's age $= x+26+3 \Rightarrow x+29$

$\therefore$ It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29) = 360$

$x^2+3x+29x+87 = 360$

$x^2+32x-273=0$ (required quadratic form)

iv) Let the speed of train be x Km/h.

speed = distance / time

$\rightarrow$ Time taken for travel 480 Km $= \frac{480}{x}$ hrs.

in the second condition,

speed of train $= (x-8)$ Km/h

Given that the train will take 3 hrs more to cover the same distance

$\therefore$ Time taken for travelling 480 Km $= \left(\frac{480}{x}+3\right)$ hrs

speed $\times$ time = distance

$(x-8)\left(\frac{480}{x}+3\right) = 480$

$\Rightarrow 480+3x-\frac{3840}{x}-24 = 480$

$3x - \frac{3840}{x} = 24$

$3x^2 - 3870 = 27x$

$3x^2 - 27x - 3870 = 0$

$\rightarrow x^2 - 9x - 1290 = 0$ (Required quadratic form)

Exercise 4.2

1. i) $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10$

$x(x-5) + 2(x-5)$

$(x-5)(x+2)$

Root of this eq. are the values for which $(x-5)(x+2) = 0$

$\therefore x - 5 = 0$ or $x + 2 = 0$

i.e.

$x = 5$ or $x = -2$

ii) $2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6$

$2x(x+2) - 3(x+2)$

$(x+2)(2x-3)$

$\Rightarrow (x+2)(2x-3) = 0$

$x = -2,\ x = \frac{3}{2}$

iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2})$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$x = \frac{-5}{\sqrt{2}},\ x = -\sqrt{2}$

iv) $2x^2 - x + \frac{1}{8} = 0$

$\frac{1}{8}(16x^2 - 8x + 1)$

$\frac{1}{8}(16x^2 - 4x - 4x + 1)$

$\frac{1}{8}(4x(4x-1) - 1(4x-1))$

$\Rightarrow \frac{1}{8}(4x-1)^2$

$\longrightarrow (4x-1)^2 = 0$

$\therefore 4x - 1 = 0,\ 4x - 1 = 0$

$x = \frac{1}{4},\ x = \frac{1}{4}$

v) $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1$

$10x(10x-1) - 1(10x-1)$

$(10x-1)^2$

$\rightarrow (10x-1)^2 = 0$

$\therefore 10x - 1 = 0,\ 10x - 1 = 0$

$x = \frac{1}{10},\ x = \frac{1}{10}$

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2. Let the number of John's marbles be x

∴ no. of Jivanti's marble $= 45 - x$

→ after losing 5 marbles,

No. of John's marbles $= x - 5$

" " Jivanti's " $= 45 - x - 5$

$\Rightarrow 40 - x$

⇒ The product of their marbles is 124.

∴ $(x-5)(40-x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

∴ factorize the eq.

$x^2 - 36x - 9x + 324 = 0$

$x(x-36) - 9(x-36)$

$(x-36)(x-9)$

$\Rightarrow x - 36 = 0,\ x - 9 = 0$

$x = 36$ or $x = 9$

∴ If the no of John's marbles = 36

∴ no. of Jivanti's marbles $= 45 - 36 \Rightarrow 9$

ii) Let the no. of toys produced be x

∴ Cost of production of each toy =

₹ $(55 - x)$

∴ Total production of toys ⇒ ₹ 750

$\Rightarrow x(55-x) = 750$

$x^2 - 55x + 750 = 0$

∴ now to factorize this eq.

→ $x^2 - 25x - 30x + 750 = 0$

$x(x-25) - 30(x-25) = 0$

$(x-25)(x-30) = 0$

$\Rightarrow x - 25 = 0,\ x - 30 = 0$

$x = 25,\ x = 30.$

∴ no of toys will be either 25 or 30.

3. Let the first no. be x and the second no. is $27 - x$.

∴ Sum of both the no. ⇒ 27

their product ⇒ $x(27-x)$

⇒ product of these no.s ⇒ 182.

$x(27-x) = 182$

$-x^2 + 27x - 182 = 0$

$\Rightarrow x^2 - 27x + 182 = 0$

by factorize we get, 13 and 14 are the no.s whose sum is 27 and product is 182.

$x^2 - 13x - 14x + 182 = 0$

$x(x-13) - 14(x-13) = 0$

$(x-13)(x-14) = 0$

$\Rightarrow x = 13,\ x = 14$

first no. ⇒ 13

other no. ⇒ $27 - 13 = 14$

⇒ If first no. ⇒ 14

other no. ⇒ $27 - 14 = 13$

⇒ The no.s are 13 and 14.

4. To find: Consecutive integers sum of whose square is 365.

→ Consecutive integers mean that the difference b/w the integers is of 1

⇒ Let the consecutive positive Integers be x and $x+1$.

→ Given that ⇒ $x^2 + (x+1)^2 = 365$

⇒ $x^2 + x^2 + 1 + 2x = 365$

⇒ $2x^2 + 2x - 364 = 0$

⇒ $x^2 + x - 182 = 0$

⇒ factorize the above Quadratic eq.

$x^2 + 14x - 13x - 182 = 0$

$x(x+14) - 13(x+14) = 0$

$(x+14)(x-13) = 0$

⇒ $x+14 = 0$, $x-13 = 0$

$x = -14$, $x = 13$.

∴ the question ask for positive integers.

x can only be 13

∴ $x+1 = 13+1 = 14$

⇒ Two consecutive positive integers will be 13 and 15.

5. Let the base of the Δ be x

→ Altitude = $x-7$

In right-angled triangle.

ACQ,

Pythagoras therem,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$

⇒ $13^2 = x^2 + (x-7)^2$

$169 - x^2 - x^2 - 49 + 14x = 0$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

⇒ $x^2 - 12x + 5x - 60 = 0$

$x(x-12) + 5(x-12) = 0$

$(x+5)(x+12) = 0$

$x = -5$ or 12.

base = 12.

altitude = $12 - 7 = 5$ cm.

∴ 5 cm and 12 cm are the two sides of the given triangle.

6. To find: No. of articles produced and cost of each article.

⇒ Let the no. of articles produced be x

∴ Cost of production of each article = $2(2x+3)$

⇒ The total cost of production = Total quantity produced × cost of one article

Given:
Total cost of production = ₹90

$\therefore x(2x+3) = 90$

$2x^2 + 3x - 90 = 0$

$2x^2 + 15x - 12x - 90 = 0$

$x(2x+15) - 6(2x+15) = 0$

$(2x+15)(x-6) = 0$

$x = \frac{-15}{2}, \; x = 6$

$\therefore$ x can only be 6.
The no. of articles produced = 6
Cost of each article = $2 \times 6 + 3$
$= 12 + 3$
$= 15$

Exercise-4.3

1. i) $2x^2 - 7x + 3 = 0$

→ Dividing by coefficient of x^2,

$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

adding and subtracting the square of $\left(\frac{b}{2}\right) = \frac{7}{4}$, we get

$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

use the formula $(a-b)^2 = a^2 + b^2 - 2ab$

$\left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$

$\left(x - \frac{7}{4}\right)^2 + \frac{24-49}{16} = 0$

$\left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$

So,

$\left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

When, $x - \frac{7}{4} = -\frac{5}{4}$

$x = \frac{-5}{4} + \frac{7}{4} = \frac{2}{4} \Rightarrow \frac{1}{2}$

When, $x - \frac{7}{4} = \frac{5}{4}$

$x = \frac{5}{4} + \frac{7}{4} = \frac{12}{4} \Rightarrow 3$

⇒ The value of x is 3 and $\frac{1}{2}$.

ii) $2x^2 + x - 4 = 0$

→ Dividing by coefficient of x^2

$x^2 + \frac{x}{2} - 2 = 0$

Adding and subtracting the square of $\frac{b}{2} = \frac{1}{4}$, we get.

$x^2 + 2(x) \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$

use the formula $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow (x+\frac{1}{4})^2 - (\frac{1}{16}+2) = 0$

$(x+\frac{1}{4})^2 = \frac{33}{16}$

$x+\frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$x = \frac{-1}{4} \pm \frac{\sqrt{33}}{4}$

$x = \frac{-1 \pm \sqrt{33}}{4}$

$\Rightarrow x = \frac{-1+\sqrt{33}}{4}$ or $x = \frac{-1-\sqrt{33}}{4}$

iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Divide the whole eq. by 4.

$\frac{4x^2 + 4\sqrt{3}x + 3}{4} = \frac{0}{4}$

add or subtract the square of the coefficient of x,

$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} + (\frac{\sqrt{3}}{2})^2 - (\frac{\sqrt{3}}{2})^2 = 0$

use the formula $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow (x+\frac{\sqrt{3}}{2})^2 + \frac{3}{4} - (\frac{\sqrt{3}}{2})^2 = 0$

$(x+\frac{\sqrt{3}}{2})^2 + \frac{3}{4} - \frac{3}{4} = 0$

$\Rightarrow x + \frac{\sqrt{3}}{2} = 0$ and $x + \frac{\sqrt{3}}{2} = 0$

$\Rightarrow x = 0 - \frac{\sqrt{3}}{2}$ and $x = 0 - \frac{\sqrt{3}}{2}$

$\Rightarrow x = -\frac{\sqrt{3}}{2}$ and $x = -\frac{\sqrt{3}}{2}$

$\therefore$ Roots of equation are $= x = (-\frac{\sqrt{3}}{2}), (-\frac{\sqrt{3}}{2})$

iv) $2x^2 + x + 4 = 0$

Divide by coefficient of x^2, we get,

$x^2 + \frac{x}{2} + 2 = 0$

adding and subtracting the square of coefficient of x,

$\Rightarrow x^2 + \frac{x}{2} + 2 + (\frac{1}{4})^2 - (\frac{1}{4})^2 = 0$

use the formula $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow (x+\frac{1}{4})^2 + 2 - (\frac{1}{4})^2 = 0$

$\Rightarrow (x+\frac{1}{4})^2 + 2 - \frac{1}{16} = 0$

$\Rightarrow (x+\frac{1}{4})^2 + \frac{32-1}{16} = 0$

$\Rightarrow (x+\frac{1}{4})^2 + \frac{31}{16} = 0$

$\Rightarrow (x+\frac{1}{4})^2 = -\frac{31}{16}$

$\rightarrow$ Square of a no. cannot be negative, hence roots of the given eq. do not exist.

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2\. i) $2x^2 - 7x + 3 = 0$

According to quadratic formula,

$ax^2 + bx + c$

$a = 2,\ b = -7,\ c = 3$

$\therefore$ The roots are $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \dfrac{[-(-7) \pm \sqrt{49 - 4 \times 2 \times 3}}{2 \times 2} = \dfrac{[7 \pm \sqrt{49 - 24}]}{4} =$

$\left(\dfrac{7 \pm \sqrt{25}}{4}\right) = \left(\dfrac{7 \pm 5}{4}\right)$

$\Rightarrow \dfrac{7+5}{4} = \dfrac{12}{4} = 3$ and $\dfrac{7-5}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

$\Rightarrow$ roots are $= 3$ and $\dfrac{1}{2}$

ii) $2x^2 + x - 4 = 0$

According to the quadratic formula;

$ax^2 + bx + c$

$\Rightarrow a = 2,\ b = 1,\ c = -4$

$\therefore$ The roots are $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \dfrac{[-1 \pm \sqrt{1 - 4(2)(-4)}}{2 \times 2} = \dfrac{-1 \pm \sqrt{33}}{4}$

$\therefore$ The roots are $= \dfrac{-1 + \sqrt{33}}{4}$ and $\dfrac{-1 - \sqrt{33}}{4}$

iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

According to quadratic formula;

$ax^2 + bx + c$

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$a = 4,\ b = 4\sqrt{3},\ c = 3$

$\therefore$ The roots are $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \left(\dfrac{-4\sqrt{3} \pm \sqrt{16 \times 3 - (4 \times 4 \times 3)}}{2 \times 4}\right) = \left(\dfrac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}\right) = \dfrac{-4\sqrt{3} \pm 0}{8}$

$= \left(-\dfrac{\sqrt{3}}{2}\right), \left(-\dfrac{\sqrt{3}}{2}\right)$

$\therefore$ The roots are $\left(-\dfrac{\sqrt{3}}{2}\right)$ and $\left(-\dfrac{\sqrt{3}}{2}\right)$

iv) $2x^2 + x + 4 = 0$

According to quadratic formula;

$ax^2 + bx + c$

$\Rightarrow a = 2,\ b = 1,\ c = 4$

$\therefore$ The roots are $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \dfrac{[-1 \pm \sqrt{1 - 4(2)(4)}\ ]}{2 \times 2} = \dfrac{-1 \pm \sqrt{1 - 32}}{4} = \dfrac{-1 \pm \sqrt{-31}}{4}$

$\Rightarrow$ This is not possible, hence the roots do not exist

3\. i) $x - \dfrac{1}{x} = 3,\ x \neq 0$

Taking L.C.M and then cross multiplying,

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -3,\ c = -1$

So, the roots are,

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{[-(-3) \pm \sqrt{(-3)^2-4(1)(-1)}]}{2}$

$\Rightarrow \frac{3 \pm \sqrt{13}}{2}$

$\therefore$ The roots are $= \frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$

ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$

$\frac{[(x-7)-(x+4)]}{[(x+4)(x-7)]} = \frac{11}{30}$

$\frac{-11}{[(x-4)(x-7)]} = \frac{11}{30}$

$(x+4)(x-7) = -30$

$x^2 - 3x - 28 = -30$

$x^2 - 3x + 2 = 0$

Factorizing the quadratic equation,
Such that the product of two no.s is 2 and their sum is 3 (2×1 and 2+1

$x(x-2) - 1(x-2) = 0$

$(x-2)(x-1) = 0$

i.e. $x-2=0$ or $x-1=0$

$\therefore$ $x=1$ and $x=2$ are the roots of the given eq.

4. Let the present age of Rehman ~~be x~~ be x years.

$\therefore$ 3 years ago, his age $= (x-3)$

5 years hence, his age will be $(x+5)$ years

Given: The sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow 3(2x+2) = (x-3)(x+5)$

$6x+6 = x^2+2x-15$

$x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3 - 21 = 0$

$x(x-7) + 3(x-7) = 0$

$x = 7, -3$

$\Rightarrow$ age cannot be negative.

$\therefore$ Rehman's present age is 7 years.

5. Let the marks in Maths be x.

$\therefore$ English $= 30 - x$

ACQ,

She got 2 marks more in Maths and 3 marks less in English.

$\therefore$ The product of their marks $= 210$

$(x+2)(30-x-3) = 210$

$(x+2)(27-x) = 210$

$\Rightarrow -x^2 + 25x + 54 = 210$

$\Rightarrow x^2 - 25x + 156 = 0 \quad \{x^2 - 12x - 13x + 156\}$

$\Rightarrow x(x-12) - 13(x-12)$

$(x-12)(x-13) = 0$

$\Rightarrow x = 12, 13$

$\Rightarrow$ If the marks in maths are 12 then marks in english = $30 - 12 = 18$

$\Rightarrow$ If the marks in maths are 13, then marks in english = $30 - 13 = 17$.

6. To find: length and breadth

Let the shorter side of the rectangle be xm

Larger side of the rect. $\Rightarrow (x+30)$m

$\Rightarrow$ By pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{perpendicular})^2$

Diagonal of rect $\Rightarrow \sqrt{l^2 + b^2}$

$\Rightarrow \sqrt{x^2 + (x+30)^2}$

Given that: diagonal of the rect. is 60m more than the shorter side.

$\therefore \sqrt{x^2 + (x+30)^2} = x + 60$

squaring both sides, we get,

$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$x^2 - 60x - 2700 = 0$

$\Rightarrow$ factorize it

$\Rightarrow x(x-90) + 30(x-90) = 0$

$(x-90)(x+30) = 0$

$x = 90, -30$

$\Rightarrow$ side cannot be negative, so,

$\Rightarrow$ length of shorter side = 90m

$\Rightarrow$ length of larger side $\Rightarrow$ 120m.

7. Let the larger and smaller no. be x and y respectively.

ACQ,

$\Rightarrow$ Difference of squares of two no.s is 180

$\Rightarrow$ The square of smaller no. is 8 times square of the larger no.

So,

$x^2 - y^2 = 180$ — (i)

$y^2 = 8x$ — (ii)

putting value of (ii) in (i)

$x^2 - 8x = 180$

$x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$x(x-18) + 10(x-18) = 0$

$(x-18)(x+10)$

$x = 18, -10$.

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$\Rightarrow$ The larger no. cannot be negative as it makes the square of smaller no. negative, which is not possible.

$\Rightarrow$ The larger no. will be 18

$x = 18$

$\therefore y^2 = 8x$

8×18

144

$y = \pm\sqrt{144} = \pm 12$ (smaller no.)

8. To find : speed of train.

Let the speed of train be x km/hr

Time taken to cover 360km $= \frac{360}{x}$ hr.

$\rightarrow$ time $= \frac{\text{distance}}{\text{speed}}$

Given that, if the speed would be 5 km/hr more the same distance would be covered in 1 hour less, i.e.

$\Rightarrow$ if speed $= x+5$

time $= \left(\frac{360}{x} - 1\right)h$

then, $d = s \times t$

$(x+5)\left(\frac{360}{x} - 1\right) = 360$

form quadratic eq:

$360 - x + \frac{1800}{x} - 5 = 360$

$360x - x^2 + 1800 - 5x = 360x$

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$x^2 + 5x - 1800 = 0$

$\Rightarrow x^2 + 45x - 40x - 1800$

$x(x+45) - 40(x+45) = 0$

$(x+45)(x-40)$

$x = -45, 40$

$\therefore$ speed of train can't be negative

So, speed is 40 km/hr.

9. Let the time taken by the smaller pipe to fill the tank be x hr.

$\Rightarrow$ Time taken by the larger pipe $= (x-10)$ hr

Part of tank filled by the larger pipe in 1 hr

$\Rightarrow \frac{1}{x-10}$

Part of tank filled by the smaller pipe in 1 hr

$\Rightarrow \frac{1}{x}$

Tank can be filled in $= \frac{75}{8}$ hr by both the pipes

$\frac{75}{8}$ hr. multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e. 1

$\frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

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$\Rightarrow \dfrac{x-10+x}{x(x-10)} = \dfrac{8}{75}$

$\Rightarrow \dfrac{2x-10}{x(x-10)} = \dfrac{8}{75}$

$\Rightarrow 75(2x-10) = 8x^2 - 80x$

$150x - 750 = 8x^2 - 80x$

$8x^2 - 230x + 750 = 0$

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$8x(x-25) - 30(x-25) = 0$

$(x-25)(8x-30) = 0$

$x = 25, \dfrac{30}{8}$

∴ Time taken by the smaller pipe cannot be $\dfrac{30}{8} = 3.75$ hrs.

⇒ The time taken by the larger pipe will be negative, which is logically not possible.

∴ Time taken by the smaller and larger pipe will be 25 and 25-10 = 15 hrs respectively.

10. Let the average speed of passenger train be x km/h.

⇒ average speed of express train = (x + 11) km/h

⇒ Time taken by express train to cover 132 km = 1 hour, which is less than the passenger train to cover the same distance.

$t = \dfrac{d}{s}$

∴ Time taken by passenger train $= \dfrac{132}{x}$

→ Time taken by express train $= \dfrac{132}{x+11}$

⇒ Time taken by express train = 1

$\therefore \dfrac{132}{x} - \dfrac{132}{x+11} = 1$

$\Rightarrow 132\left[\dfrac{x+11-x}{x(x+11)}\right] = 1$

$\Rightarrow \dfrac{132 \times 11}{x(x+11)} = 1$

$\Rightarrow 132 \times 11 = x(x+11)$

$\Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow x^2 + 44x - 33x - 1452 = 0$

$x(x+44) - 33(x+44) = 0$

$(x+44)(x-33) = 0$

$\Rightarrow x = -44, 33$

∵ Speed cannot be negative.

⇒ The speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

11. Let the sides of the two squares be x m and y m.

$\therefore$ their perimeter = $4x$ and $4y$ resp.
their areas = x^2 and y^2 ″

It is given that $4x - 4y = 24$ [diff. of perimeter].
or $x - y = 6$
$x = y + 6$
also,
$x^2 + y^2 = 468$ [Sum of squares is 468]

$(6+y^2) + y^2 = 468$
$36 + y^2 + 12y + y^2 = 468$
$2y^2 + 12y - 432 = 0$
$y^2 + 6y - 216 = 0$
$y^2 + 18y - 12y - 216 = 0$
$y(y+18)(y-12) = 0$
$y = -18, 12$
Side of square cannot be negative.
$\Rightarrow$ The sides of square are 12m and $(12+6)m = 18$ m.

Exercise - 4.4

1. There are three types of roots that are possible for a quadratic eq:-

i) $2x^2 - 3x + 5 = 0$
$\Rightarrow$ Comparing this eq. with $ax^2 + bx + c = 0$,
$a = 2, b = -3, c = 5$

Discriminant $D = b^2 - 4ac = (-3)^2 - 4(2)(5)$
$= 9 - 40 = -31$

as, $b^2 - 4ac < 0$,
$\therefore$ no real root is possible for the given eq.

ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
$a = 3, b = -4\sqrt{3}, c = 4$
$\Rightarrow b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$
Discriminant $= 48 - 48 = 0$
as, $b^2 - 4ac = 0$

$\therefore$ real roots exist for the given eq. and they are equal to each other.

$\Rightarrow$ roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$

$\frac{-b}{2a} = \frac{-(4\sqrt{3})}{2\times3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

$\therefore$ the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$

~~iii) $\sqrt{3}x^2 + 4\sqrt{3}x + 4 = 0$~~

iii) $2x^2 - 6x + 3 = 0$

$a = 2$

$b = -6$

$c = 3$

→ Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

$\Rightarrow 36 - 24 = 12.$

as, $b^2 - 4ac > 0$

→ distinct real roots exist for this eq. as follows,

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$

$= \dfrac{6 \pm \sqrt{12}}{4} = \dfrac{6 \pm 2\sqrt{3}}{4}$

$= \dfrac{3 \pm \sqrt{3}}{2}$

So, the roots are $\dfrac{3+\sqrt{3}}{2}$ or $\dfrac{3-\sqrt{3}}{2}$

2. i) $2x^2 + kx + 3 = 0$

comparing eq. with $ax^2 + bx + c = 0$,

$a = 2,\ b = k,\ c = 3$

Discriminate $= b^2 - 4ac = (k)^2 - 4(2)(3) =$

$\Rightarrow k^2 - 24$

for equal roots,

Discriminant $= 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

ii) $kx(x-2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

comparing this eq. with $ax^2 + bx + c = 0$,

$a = k,\ b = -2k,\ c = 6.$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k = 0$

$= 4k(k-6) = 0$

either $4k = 0$ or $k = 6$

$\Rightarrow k = 0,\ k = 6.$

3. Let the breadth of mango grove be l.

length of mango grove will be $2l$.

$\Rightarrow$ Area of mango grove $= (2l)(l) = 2l^2$

$2l^2 = 800$

$\Rightarrow l^2 - 400 = 0$

$\Rightarrow l^2 = 400$

$l = \pm 20$

$\Rightarrow$ length cannot be negative.

$\therefore$ breadth of mango grove $= 20m$

length " " " $= 2 \times 20 = 40m$

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4. Let the age of one friend be x years.
Age of other friend $= (20-x)$ years

$\Rightarrow$ 4 years ago,
age of 1st friend $= (x-4)$ years.
age of 2nd friend $= (20-x-4) = (16-x)$ y

Given that,
$(x-4)(16-x) = 48$
$16x - 64 - x^2 + 4x = 48$
$x^2 - 20x + 112 = 0$
$\Rightarrow a = 1, b = -20, c = 112.$

discriminant $\Rightarrow b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448$
$= -48.$

as, $b^2 - 4ac < 0$
$\therefore$ no real root is possible for this eq. and hence, this situation is not possible

5. Let the length and breadth of the park be l and b.
$\rightarrow$ Perimeter $= 2(l+b) = 80$
$l + b = 40$ or, $b = 40 - l$
$\rightarrow$ area $= l \times b = l(40-l)$
$\Rightarrow 40l - l^2 = 400$ (given)
$l^2 - 40l + 400 = 0$
$\rightarrow$ comparing this eq. with $al^2 + bl + c = 0$,
$a = 1,$
$b = -40, c = 400$

Discriminant $D = b^2 - 4ac = (-40)^2 - 4(1)(400)$
$= 1600 - 1600 = 0$
as,
$b^2 - 4ac = 0$
$\therefore$ this eq. has equal real roots and hence, this situation is possible.

$\Rightarrow$ Root of this eq,
$l = \frac{-b}{2a}, l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$

$\Rightarrow$ length of park, $l = 20m$
breadth " " , $b = 40 - l = 40 - 20 = 20m$

13/07/19

Notes:-
$\Rightarrow$ Standard form of quadratic eq:-
$ax^2 + bx + c = 0$
where, a, b and c are constant, $a \neq 0$.

- Roots $\Rightarrow$ Value of variable which satisfy the eq.
- quadratic eq. has two roots.
- Let α and β are two roots of quadratic eq. $\Rightarrow ax^2 + bx + c = 0$, then $\Rightarrow \alpha + \beta = \frac{-b}{a}$
$\alpha\beta = c$
- If α and β are the roots of quadratic eq. then $\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$